10.9 POLAR CONICS

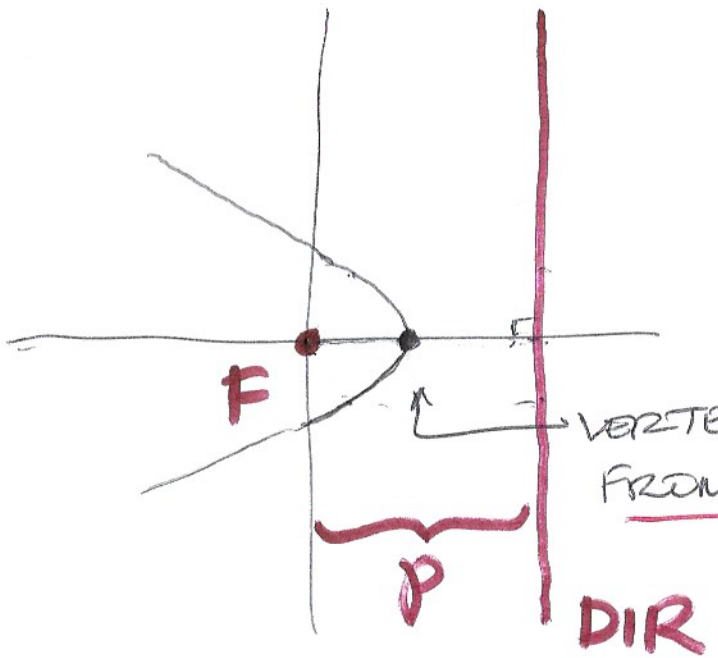
DEF'N: A CONIC IS THE LOCUS OF POINTS IN A PLANE
SUCH THAT
THE RATIO OF THE DISTANCE BETWEEN A POINT (ON THE CONIC)
TO A FIXED POINT ("FOCUS")
TO THE DISTANCE BETWEEN THAT POINT
TO A FIXED LINE ("DIRECTRIX")
(NOT CONTAINING THE FOCUS)
IS A FIXED CONSTANT ("ECCENTRICITY")
↑ e ← NOT THE e FROM e^x, $\ln x$...

ELLIPSE	$0<e<1$	← AS $e \to 0$, CONIC → CIRCLE
PARABOLA	$e=1$	
HYPERBOLA	$e>1$	

POLAR EQUATION OF CONIC WITH FOCUS @ POLE AND DIRECTRIX (VERTICAL) p UNITS TO RIGHT OF FOCUS

p ← DISTANCE BETWEEN FOCUS + DIRECTRIX

$$r = \frac{ep}{1+e\cos\theta}$$

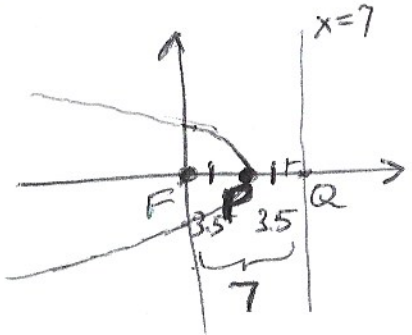


VERTEX LIES ON PERPENDICULAR ← AXIS OF SYMMETRY FROM FOCUS TO DIRECTRIX

CONIC ALWAYS WRAPS AROUND FOCUS
AWAY FROM DIRECTRIX
NEVER INTERSECTS DIRECTRIX

FIND THE POLAR EQ'N OF CONIC WITH FOCUS @ POLE AND DIRECTRIX TO THE RIGHT OF THE POLE

① IF CONIC IS PARABOLA ← $e=1$ WITH DIRECTRIX $x=7$

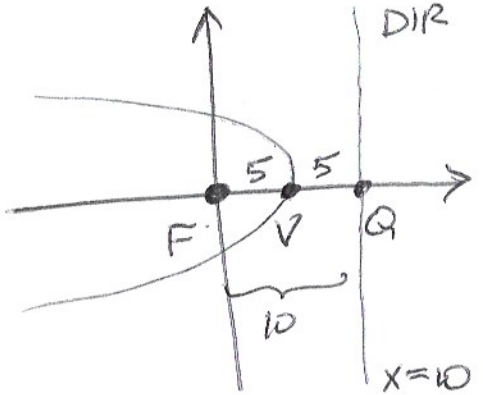


$$r = \frac{ep}{1+e\cos\theta} = \frac{1\cdot 7}{1+1\cdot\cos\theta} = \frac{7}{1+\cos\theta}$$

$p=7,\ e=1 = \frac{PF}{PQ}$

VERTEX = (3.5, 0)

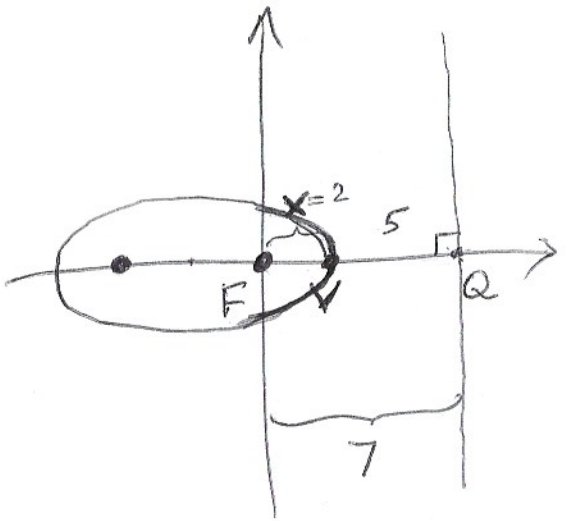
(2) CONIC IS PARABOLA WITH VERTEX $(5,0)$



$$r = \frac{ep}{1+e\cos\theta} = \frac{1\cdot 10}{1+1\cdot\cos\theta} = \frac{10}{1+\cos\theta}$$

$e = 1,\ p = 10$

(3) CONIC IS ELLIPSE WITH ECCENTRICITY $\frac{2}{5}$ AND DIRECTRIX $x = 7$



FOCUS OF ELLIPSE IS NEVER THE CENTER OF ELLIPSE

$$r = \frac{ep}{1+e\cos\theta} = \frac{\frac{2}{5}\cdot 7}{1+\frac{2}{5}\cos\theta}\cdot\frac{5}{5} = \frac{14}{5+2\cos\theta}$$

$e = \frac{2}{5},\ p = 7$

$$\frac{PF}{PQ} \quad \frac{VF}{VQ} = \frac{2}{5}$$

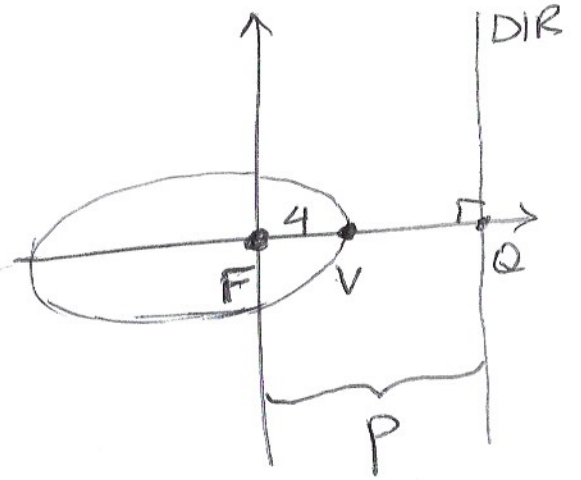
$$\frac{x}{7-x} = \frac{2}{5}$$

$5x = 2(7-x) = 14-2x$

$7x = 14 \longrightarrow x = 2$

(4) CONIC IS ELLIPSE WITH ECCENTRICITY $e = \frac{2}{3}$

AND VERTEX (4,0)



$$r = \frac{ep}{1+e\cos\theta} = \frac{\frac{2}{3}\cdot 10}{1+\frac{2}{3}\cos\theta}\cdot\frac{3}{3} = \frac{20}{3+2\cos\theta}$$

$$e = \frac{2}{3} = \frac{VF}{VQ} = \frac{4}{p-4}$$

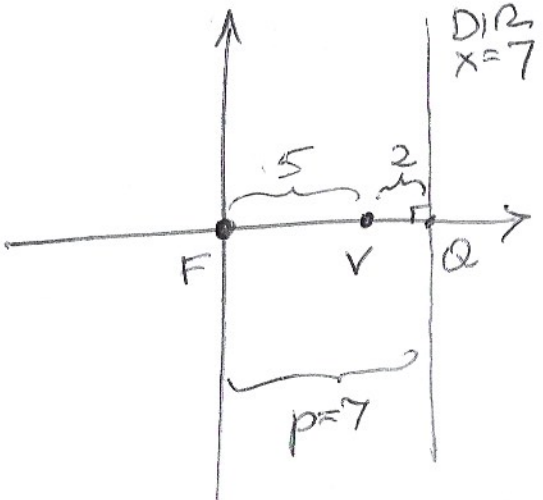
$$\frac{2}{3} = \frac{4}{p-4}$$

$$2(p-4) = 12$$

$$p-4 = 6$$

$$p = 10$$

(5) CONIC WITH VERTEX (5,0) AND DIRECTRIX $x = 7$



$$r = \frac{ep}{1+e\cos\theta} = \frac{\frac{5}{2}\cdot 7}{1+\frac{5}{2}\cos\theta}\cdot\frac{2}{2} = \frac{35}{2+5\cos\theta}$$

$p = 7$

$$e = \frac{VF}{VQ} = \frac{5}{2}$$ → CONIC IS HYPERBOLA ($e > 1$)

~~(6) CONIC WITH~~

POLAR EQUATION OF CONIC

WITH FOCUS @ POLE + DIRECTRIX p UNITS FROM FOCUS